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Theory of weakly coupled relativistic plasma: diffusion and transport across a magnetic field

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Abstract. Explicit analytic calculations concerning transport coefficients are presented for a relativistic magnetized and collisional plasma, especially at moderately-relativistic (10 keV $\leqslant kT \leqslant$ 1 GeV) and ultra-relativistic ($kT >$ 1 GeV) temperature limits covering up to Planck's scale. Transport coefficients, such as electrical resistivity (η_R) and thermal conductivity (κ_R), except the thermoelectric coefficient (λ_R), however, strikingly decrease with increase in thermal energy (kT), particularly in a moderately-relativistic temperature limit, whereas in an ultra-relativistic temperature regime the trend is similar, although it is rather more drastic. Our results are of importance for understanding collisional transport processes occurring in astrophysical situations and laboratory diagnostic experiments.

1. Introduction

Small angle binary collisions of long-range inverse square types of force have been formulated to some extent [1–3]. They largely emphasize the importance of the Fokker–Planck collisional term in the fluid description of relativistic magnetoplasma. However, the specific solution of the relativistic Boltzmann transport equation or Fokker–Planck equation, to derive various transport coefficients of interest in appropriate temperature limits, has never been attempted. Van Erkelens and Van Leeuwen [4] formulated the relativistic transport equation in covariant forms and analytically predicted transport coefficients in the form of manifold I-integrals. Likewise, Gottal and Gaviel [5] discussed the covariant description of the behaviour of the test particle in a relativistic plasma and have gone on at length to show that binary collisions do contribute significantly to diffusion tensors. Their analysis illustrates that in an anisotropic and totally-ionized plasma the Cerenkov emission is hardly noticed. Furthermore, Van Leeuwen *et al* [6], while addressing the relativistic kinetic theory in covariant form relevant to cosmology, have revealed that the collision operator is independent of the magnetic field. In the same manner, their Coulomb logarithm term appears to be independent of the magnetic field or gyrofrequency, to explain the astrophysical and magnetized plasma phenomena.

In this paper, we report purportedly the specific solutions of the scattered flux integrals inherent in the relativistic Boltzmann transport equation and the consequent formulations encountered in binary collisions concerning a fully-ionized, singly-charged, two component (electron, ion) plasma species, having diffusion across a magnetic field $\boldsymbol{B}(0, 0, B)$ [7]. The novel features of the methodology, relying on relativistic formulation and analytic evaluation of the collision scattered flux integrals in terms of modified Bessel functions, and its recursion do represent significant innovation on technicalities. In addition, the

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definite improvements of our work include the features concerning the relativistic and ultra-relativistic results on the transport coefficients. We equally incorporate physical explanations of new features of the above transport coefficients in the temperature limits of interest. To be precise, the Coulomb logarithm feature has been considered properly to account for the magnetic field strength limits.

It has been exemplified that our formula for electrical conductivity qualitatively approaches that of Li and Petrasso [8] in the vanishing limit of the reciprocal of $\ln \Lambda_b$ suitable for weakly-coupled plasmas. The classical expression is easily recovered by introducing the appropriate mass ratio approximation in the collision frequency, i.e. the ions equilibrate among themselves at a rate $\tau_{ii}^{-1} \ll \tau_{ee}^{-1}$ and hence the term $(m^-/m^+)^{1/2}$ appears and leads to the appropriate electrical conductivity. However, owing to their Fokker–Planck formalism, which includes cumulative small angle collision effects in a non-magnetic electron plasma, numerical mismatch is somewhat apparent. The more interesting and innovative attribute of the present analysis is that it clearly derives the diffusion transport coefficients suitable for all temperature regimes, i.e. non-relativistic, weakly- or moderately-relativistic and ultra-relativistic thermal regimes, in the form of a complete analytic evaluation for the first time. Ameliorating the early analyses, which do not describe appropriately the quantized and relativistic formulation, we first deal with the problem valid for a fully-ionized plasma, in a semiclassical treatment relevant to the relativistic theory. We further propose to extend the present work to include correct quantum plasma analysis, by way of a quantized relativistic Dirac equation or the correct Lagrangian formulation, to account for the plasma in high-energy elementary particle processes.

The present kinetic model is intrinsically relevant to the study of plasmas especially in moderately-relativistic and ultra-relativistic temperature limits. The former limit can describe plasmas commonly encountered in fusion devices, while the latter case is relevant to astrophysical situations including interplanetary space plasmas, radiogalaxies, pulsar winds or jets, pulsar magneto-spheres, sporadically exploding black holes and the ultra-relativistic plasma particles linked with the Big-Bang explosion covering Planck's scale.

In our relativistic collisional kinetic model we choose an equilibrium distribution function $f_0^{\pm}$ valid for a weakly coupled (ion, electron) plasma [2, 4] and an appropriate Coulomb logarithm $(\log(2/\theta_0) = \log(\lambda_D/\lambda_C)$ independent of magnetic field. Here λ_D is the conventional Debye length and λ_C is the corresponding classical distance of closest approach. However, for a strong magnetic field, we include the magnetic field term (B) in the Coulomb logarithm expression, as prescribed in Bernstein and Baxter [2], and, likewise, electrostatic shielding is equally taken into account [9, 10].

We discuss the dependence of the transport coefficients on the strength of the magnetic field embedded in the modified Coulomb logarithm both qualitatively and quantitatively [2, 9, 10]. Our analysis reveals that for low density plasma ($N = 10^{13}$ cm^{-3})$\lambda_D/\lambda_C < \lambda_G/\lambda_C$ only when $B < 10^4$ G, which can be very easily regarded as a weak or moderate field. Note that $\lambda_G (= (kT/m)^{1/2}(mc/eB))$ symbolizes the mean gyration radius of the particle species. Thus for a low-density plasma both in a weak and moderate magnetic field the Coulomb logarithm is independent of magnetic field term. However, in the case of the strong magnetic field, i.e. $B \geqslant 10^4$ G, it is imperative to take an appropriately modified Coulomb logarithm as discussed below. In a strong magnetic field the ratio $\lambda_D/\lambda_C > \lambda_G/\lambda_C$ and consequently we include the Coulomb logarithm $\log \lambda_G/\lambda_C$, which is dependent on the magnetic field. In addition, following Schram [9] and O'Neil [10], we also equally include the appropriate Coulomb logarithm to account for electrostatic shielding $(\log \lambda_D/\lambda_C)$ in a strong magnetic field. The prefactor as indicated in equation (B.4) of [10] is already contained in our collision integral.

Our unique results reveal that in the moderately-relativistic temperature regime, the electrical resistivity and thermal conductivity register a diminishing profile with increasing temperature, and these quantities increase significantly with the increase in the dimensionless physical quantities z and z' [11–14]. However, in the ultra-relativistic temperature limits these coefficients behave similarly, but the changes are more drastic. Note that our formula for electrical conductivity in the limit of moderately-relativistic temperature only approaches that of Van Erkelens and Van Leeuwen [15] by putting $n_3 = 0$ in their ternary model. In addition, we derive specific ready-to-use formulae for the $\boldsymbol{E} \times \boldsymbol{B}$ drift velocity, electrical resistivity, thermoelectric coefficient and thermal conductivity for all the three important temperature regimes. We believe that the present paper embodies all thermal regimes that have significant geophysical and global applications of natural plasmas, and the explicit binary interaction phenomena in both weak and strong magnetic field limits, for the first time. Our unified formalism covering all regimes definitely demonstrates some sort of superiority, in a realistic and observational sense, of laboratory diagnostics and simulations.

The paper is organized as follows. Section 2 contains the kinetic equations and the relativistic formulation leading to appropriate calculations, whereas in section 3, the transport coefficients are derived analytically. Section 4 and 5 relate to the limiting thermal regimes: non-relativistic and weakly- or moderately-relativistic approximations and ultra-relativistic limits, respectively.

2. Kinetic equation

A fully-ionized, singly-charged, two-component (electron, ion) relativistic collisional plasma confined in a magnetic field $\boldsymbol{B}$ can be described by the relativistic Boltzmann transport equation [2]

$$\frac{\partial f^{\pm}}{\partial t} + \frac{\boldsymbol{v}}{\gamma} \cdot \nabla f^{\pm} + \nabla_v \cdot \left[\pm \frac{e}{m^{\pm}} \left(\boldsymbol{E} + \frac{\boldsymbol{V} \times \boldsymbol{B}}{\gamma c} \right) \right] f^{\pm} = C^{\pm\pm} + C^{\pm\mp} \tag{1}$$

where the index '+' stands for ion species and '−' stands for electron species. The relativistic factor is given by $\gamma = (1 + (V^2/C^2))^{1/2}$, $\boldsymbol{v} = \boldsymbol{p}/m$. Furthermore the Boltzmann collision integral C's are given in the form,

$$C^{+,-} = -\int \mathrm{d}^3 v' \frac{1}{\gamma} |v - v'| \int \mathrm{d}\Omega \sigma_{\mathrm{sc}}^{+,-} [f^{+}(\boldsymbol{x}, \boldsymbol{v}, t) f^{-}(\boldsymbol{x}, \boldsymbol{v}', t) - f^{+}(\boldsymbol{x}, \boldsymbol{v}_1, t) f^{-}(\boldsymbol{x}, \boldsymbol{v}_1', t)]. \tag{2}$$

Similar expressions can also be written for $C^{-,+}$ and C^{--} accordingly. Note that the $\sigma_{\mathrm{sc}}^{+,-}$ stands for the Moller scattering cross section, which can be defined by

$$\sigma_{\mathrm{sc}} = \frac{e^4}{4\gamma^2 m^2 u^4} \frac{1}{\sin^4(\theta/2)}$$

where the reduced mass is defined: $m = m^+ m^- /(m^+ + m^-)$ and the relative velocity $\boldsymbol{u} = \frac{\boldsymbol{v}}{\gamma} - \frac{\boldsymbol{v}'}{\gamma}$. In a conventional way [2] $\boldsymbol{v}$ is defined as the momentum divided by the rest mass, and v, v' appearing in the bracketed terms of equation (2) correspond to the velocities of the colliding particles, whereas v_1, v_1' correspond to the velocities of the product particles.

It is further instructive to choose a properly normalized equilibrium distribution function in relativistic Maxwellian form [2],

$$f_0^{\pm} = \left(\frac{N}{2}\right) \frac{m^{\pm}}{4\pi ckT K_2(z^{\pm})} \exp\left[-z^{\pm} \left(1 + \frac{p_{\pm}^2}{m^{\pm 2} c^2}\right)^{1/2} \right] \tag{3}$$

where the plasma is chosen under isothermal ($T_+ \simeq T_- = T$) and quasineutrality conditions ($N^+ \simeq N^- = 1/2N = \int f_0^{\pm} \mathrm{d}^3 v$). $K_2(z^{\pm})$ is a Bessel function of the second kind with argument $z^{\pm}$ where $z^{\pm} = m^{\pm}c^2/kT$. In addition to this, we assume that, conventionally, the collision frequency τ^{-1} is much smaller than the relativistic gyration frequency

$$\omega^{\pm} = eB/\gamma m^{\pm} c.$$

The magnetic field $\boldsymbol{B}$ is taken along the z-direction. The electric field $\boldsymbol{E}$ is induced owing to the motion of the diamagnetic plasma. While we choose $\boldsymbol{E}$ along the y-direction, there exists a small temperature gradient $\nabla(kT)$ and a small density gradient $\nabla(N)$ along the x-direction. It is further instructive to introduce the total zero-order pressure as

$$P = \sum_{+,-} \int \mathrm{d}^3 v m^{\pm} \frac{v_i v_j}{\gamma} f_0^{\pm}$$

which on evaluation in velocity space yields $P = NkT$.

Further, following Schram [9] and O'Neil [10], we include both the strong magnetic field conditions ($B \geqslant 10^4$ G) and the Debye shielding criteria and modify the Coulomb logarithm accordingly. Thus, in the case of a strong magnetic field, equation (2) needs to include, for sake of evaluation, the following: (i) a prefactor containing $\log(\lambda_{\mathrm{G}}/\lambda_{\mathrm{c}})$; and likewise to account for the other parallel part for electrostatic shielding, (ii) an approximation leading to $\log(\lambda_{\mathrm{D}}/\lambda_{\mathrm{c}})$.

The distribution function f is expanded in a familiar way [7, 12, 13] in terms of two small parameters $\alpha(= \lambda_{\mathrm{G}}(1/N)\partial N/\partial x)$ and $\beta(= 1/\omega\tau)$, as $f = f_0 + f_1$ and $f_1 = f_0 + f_{11}$. Regarding a collision as a perturbation to zero order in β, the transport equation for ion species takes the form [9]

$$\frac{1}{\gamma} v_j \frac{\partial f_0^+}{\partial x_j} + \frac{e}{\gamma m^+ c} \epsilon_{ijk} v_j B_k \frac{\partial f_{10}^+}{\partial v_i} = 0 \tag{4}$$

which yields

$$f_{10}^+ = \frac{1}{\omega^+ \gamma} f_0^+ v_y \frac{\partial}{\partial x} \log f_0^+ . \tag{5}$$

To a first order in α and first order in β, equation (1) reduces to

$$\frac{e}{m^+} E_j \frac{\partial f_0^+}{\partial v_j} + \frac{e}{\gamma m^+ c} \epsilon_{ijk} v_j B_k \frac{\partial f_{11}^+}{\partial v_i} = C_{10}^{++} + C_{10}^{+-} \tag{6}$$

where the linearized collision integral C_{10}^{+-} is written as

$$C_{10}^{+-}(\boldsymbol{x}, \boldsymbol{v}, t) = -\int \mathrm{d}^3 \boldsymbol{v}' |\boldsymbol{v} - \boldsymbol{v}'| \int \mathrm{d}\Omega \sigma_{\mathrm{sc}}^{+-} [f_0^+(\boldsymbol{x}, \boldsymbol{v}, t) f_{10}^-(\boldsymbol{x}, \boldsymbol{v}', t) + f_{10}^+(\boldsymbol{x}_1, \boldsymbol{v}, t)$$
$$\times f_0^-(\boldsymbol{x}, \boldsymbol{v}', t) - f_0^+(\boldsymbol{x}, \boldsymbol{v}_1, t) f_{10}^-(\boldsymbol{x}, \boldsymbol{v}_1', t) - f_{10}^+(\boldsymbol{x}, \boldsymbol{v}_1, t) f_0^-(\boldsymbol{x}, \boldsymbol{v}_1', t)]. \tag{7}$$

Similar expressions can also be written for C_{10}^{++} and C_{10}^{--}. Next, we add these two expressions, i.e. C_{10}^{+-} and C_{10}^{++}, and rearrange them by using equations (5) and (3) to get

$$C_{10}^{+-} + C_{10}^{++} = -\frac{1}{\gamma} \frac{f_0^+(v)}{\omega^+ v^2} v_y \psi$$

where the symbol ψ denotes the velocity space integral as defined by

$$\psi = \frac{v}{\gamma m^+} \left[\frac{\partial}{\partial x} \log\left(\frac{N}{kT}\right) \int \mathrm{d}^3 v' |\boldsymbol{v} - \boldsymbol{v}'| \left\{ f_0^-(v') \int \mathrm{d}\Omega \sigma_{\mathrm{sc}}^{+-} \Delta(m^+ \boldsymbol{v} - m^- \boldsymbol{v}') \right.\right.$$

$$+f_0^+(v')\int \mathrm{d}\Omega\sigma_{\mathrm{sc}}^{++}\Delta(m^+v+m^-v')\bigg\}$$
$$-\frac{\partial}{\partial x}\left(\frac{1}{kT}\right)\int \mathrm{d}^3v'|v-v'|\bigg\{f_0^-(v')\int \mathrm{d}\Omega\sigma_{\mathrm{sc}}^{+-}\Delta(m^{+2}c^2\gamma v-m^{-2}c^2\gamma v')$$
$$+f_0^+(v')\int \mathrm{d}\Omega\sigma_{\mathrm{sc}}^{++}\Delta(m^{+2}c^2\gamma v+m^{+2}c^2\gamma v')\bigg\}\bigg]_y.$$

On further substituting the above value of ψ in equation (6) and evaluating with the appropriate normalized relativistic equilibrium distribution function (3), it is straightforward to obtain the perturbed part of the distribution function as follows:

$$f_{11}^+=\frac{m^+c}{kT}\frac{E_y}{B}v_xf_0^++\frac{v_x}{\gamma v^2\omega^{+2}}\bigg[\left\{\frac{1}{m^+}\frac{\partial}{\partial x}\left(\frac{1}{kT}\right)\right\}v\cdot\int \mathrm{d}^3u|u|f_0(u)\int \mathrm{d}\Omega\sigma_{\mathrm{sc}}m^{+2}c^2\gamma\Delta u$$
$$+\frac{1}{m^+}\left\{\frac{\partial}{\partial x}\log\frac{N}{T}\right\}v\cdot\int \mathrm{d}^3u|u|f_0(u)\int \mathrm{d}\Omega\sigma_{\mathrm{sc}}m^+\Delta u$$
$$+\frac{1}{m^+}\left\{\frac{\partial}{\partial x}\left(\frac{1}{kT}\right)\right\}v\cdot\int \mathrm{d}^3u|u|f_0(u)\int \mathrm{d}\Omega\sigma_{\mathrm{sc}}m^+m^-\gamma c^2\Delta u\bigg]f_0^+ \tag{8}$$

where u represents the relative velocity of the system of interacting particles and, as usual [7], $\Delta[\,]$ stands for the symbol

$$\Delta A=A-A'. \tag{9}$$

3. Transport coefficients

On taking the velocity moment and solving the appropriate integral in the centre of mass and relative velocity frame we find the collisional diffusion drift velocity u_d in the form

$$u_\mathrm{d}=c\frac{E\times B}{B^2}-\eta_\perp\frac{c^2}{B^2}M(z,z')\nabla p+\frac{3}{16}\eta_\perp R(z,z')z\frac{Nc^2}{B^2}\nabla(kT) \tag{10}$$

where

$$\eta_\perp=\frac{8}{3}\left(\frac{\pi}{2}\right)^{1/2}\frac{m^{-1/2}}{(kT)^{3/2}}e^2\log\left(\frac{2}{\theta_0}\right). \tag{11}$$

We introduce and define the following:

$$M(z,z')=\frac{K_{3/2}(z)K_0(z')}{K_2(z)K_2(z')} \tag{12}$$

and

$$R(z,z')=\frac{K_{5/2}(z)K_0(z)}{K_2(z)K_2(z)}+2z^{-1}\frac{K_{3/2}(z)K_0(z')}{K_2(z)K_2(z')}+\frac{K_{5/2}(z)K_0(z')}{K_2(z)K_2(z')} \tag{13}$$

with $z\equiv z^+$ and $z'\equiv z^-$.

The first term of equation (10) is the well known $E\times B$ drift velocity

$$V_\mathrm{D}=C\frac{E\times B}{B^2}. \tag{14}$$

Note that while evaluating the above results we have made use of the following standard integrals and recurrence relation connecting derivatives of $K_n(z)$ [14],

$$\int_0^\infty \exp[-z\cosh\theta]\cosh n\theta\,\mathrm{d}\theta=K_n(z) \tag{15}$$

$$\int_0^\infty \exp[-z\cosh\theta]\sinh^{2n}\theta\,\mathrm{d}\theta=\frac{1}{\pi^{1/2}}\left(\frac{2}{z}\right)^n\Gamma(n+\tfrac{1}{2})K_n(z) \tag{16}$$

and

$$z\frac{\mathrm{d}}{\mathrm{d}z}K_n(z) - nK_n(z) = -zK_{n+1}(z). \tag{17}$$

Following the usual classical Spitzer method [7, 12, 13], we obtain an analytic expression for the current density $\boldsymbol{J}$:

$$\boldsymbol{J} = \frac{1}{\eta_{\perp \mathrm{R}}}\boldsymbol{E}' - \lambda_{\mathrm{R}}\boldsymbol{\nabla}(kT) \times \boldsymbol{B} \tag{18}$$

where the modified EMF

$$\boldsymbol{E}' = \boldsymbol{E} + \frac{1}{c}\boldsymbol{u}_{\mathrm{d}} \times \boldsymbol{B} \tag{19}$$

and the electrical resistivity

$$\eta_{\perp \mathrm{R}} = \frac{8}{3}M(z, z')\left(\frac{\pi}{2}\right)^{1/2}\frac{m^{-1/2}}{(kT)^{3/2}}e^2 \log\left(\frac{2}{\theta_0}\right). \tag{20}$$

It is imperative to note here that Van Erkelens and Van Leeuwen [15] obtained expressions for electrical conduction coefficients of ternary and quaternary mixtures in terms of complicated vectorial brackets expressed in integral forms. However, by substituting $n_3 = 0$ in the equation (3.24) of Van Erkelens and Van Leeuwen, one can recover the value of the electrical conductivity which is analogous to that in the present paper, and recognized by Braams and Karney [11]. Klimov *et al* [16] derived an expression for relativistic beam conducting of argon plasma $\sigma(\boldsymbol{r}, t)$ in terms of beam current density $j_{\mathrm{b}}(\boldsymbol{r}, t)$ and plasma current density $j_{\mathrm{p}}(\boldsymbol{r}, t)$ in integral form only. However, specific conclusive results for fully-ionized relativistic plasma have not been attempted. The result (20) is significantly evaluated to yield the conductivity of a fully-ionized, two-component relativistic plasma diffusing across a magnetic field over a broad range of temperature.

The thermoelectric coefficient λ_{R} appearing in (18) is given by

$$\lambda_{\mathrm{R}} = \frac{3}{16}z\frac{R(z, z')}{M(z, z')}\frac{Nc}{B^2}. \tag{21}$$

The heat flow vector is calculated in the usual classical Spitzer way [7, 12, 13] to the first order in α and β, in the direction of gradients and relative to the frame $\boldsymbol{u}_{\mathrm{d}}$ as

$$\boldsymbol{Q} = -\kappa_{\mathrm{R}}\boldsymbol{\nabla}(kT) + \frac{3}{4}\frac{M(z, z')}{R(z, z')}\left(\frac{kT}{m^+c^2}\right)kT\lambda_{\mathrm{R}}\boldsymbol{E} \times \boldsymbol{B} \tag{22}$$

with the thermal conductivity κ_{R} given by

$$\kappa_{\mathrm{R}} = \frac{1}{16}\left(\frac{2m^+}{m^-}\right)^{1/2}\frac{R(z, z')}{M(z, z')}z\frac{N^2c^2}{B^2}kT\eta_{\perp \mathrm{R}}. \tag{23}$$

It is observed that the dominant contribution to the thermal conductivity is from ion–ion collisions.

Further, we employ desired approximations [17, 18] for $K_n(z)$ relevant to non-relativistic, $(z, z' \geqslant 1)$ moderately-relativistic $(z, z' \gg 1)$ and ultra-relativistic $(z, z' \ll 1)$ temperature regimes in the above results and obtain corresponding expressions for different transport coefficients, valid for different temperature regimes.

4. Non-relativistic and weakly-relativistic approximations

In the non-relativistic temperature regime, i.e. when $z, z' \gg 1$ and $K_n(z)$ is approximated as $K_n(z) \simeq (\pi/2z)^{1/2}\exp(-z)$, equations (12) and (13) reduce to $M(z, z') \simeq 1$ and $R(z, z') \simeq 4$, respectively. On further combining these results in equations (20), (21) and (23), it is straightforward to recover the classical results of transport coefficients, i.e. the electrical resistivity

$$\eta_\perp = \frac{8}{3}\left(\frac{\pi}{2}\right)^{1/2}\frac{m^{-1/2}}{(kT)^{3/2}}e^2\log\left(\frac{2}{\theta_0}\right) \tag{24}$$

the thermoelectric coefficient

$$\lambda = \frac{3}{4}\frac{Nc^2}{B^2} \tag{25}$$

and the thermal conductivity

$$\kappa = \frac{1}{4}\left(\frac{2m^+}{m^-}\right)^{1/2}\frac{N^2c^2}{B^2}kT\eta_\perp. \tag{26}$$

These results are applied to the plasmas which occur in laboratory diagnostic problems. In nature, these are also abundantly visible in ionospheric plasmas, solar coronas, flares or shock waves and so on.

To deal with a situation arising from a plasma having a weakly- or moderately-relativistic temperature regime we consider the electrons to be relativistic ($z' \geqslant 1$) while the ions still retain the classical character ($z \gg 1$). The approximated value of the Bessel function of second kind $K_n(z')$ with argument z' approximates the limit, i.e.

$$K_n(z') \simeq \left(\frac{\pi}{2z'}\right)^{1/2}\exp(-z')\left[1 + \frac{4n^2-1}{8z'} + \frac{(4n^2-1^2)(4n^2-3^2)}{2!(8z')^2}\right].$$

On employing the above formulae, equations (12) and (13) are easily reduced to the simplified forms as follows:

$$M(z, z') = \left(\frac{\pi}{2}\right)^{1/2}\frac{\exp(-z')}{K_2(z')}\left(\frac{z'^2 - 0.125z' + 0.07}{z'^{5/2}}\right) \tag{27}$$

and

$$R(z, z') = 1 + \{2z^{-1} + 1\}\left(\frac{\pi}{2z'}\right)^{1/2}\frac{\exp(-z')}{K_2(z')}\left[\frac{z'^2 - 0.125z' + 0.07}{z'^{5/2}}\right] \tag{28}$$

respectively.

On further combining the above expressions with equations (20), (21) and (23), we can derive explicit formulae concerning the relevant transport coefficients in terms of the dimensionless constants z and z' to be valid for the plasmas governed by a weakly- or moderately-relativistic temperature limit.

The electrical resistivity, thus, assumes the form:

$$\eta_{\perp R} = \frac{4}{3}\pi\left[\frac{z'^2 - 0.125z' + 0.07}{z'^{5/2}}\right]\frac{\exp(-z')}{K_2(z')}\frac{m^{-1/2}}{(kT)^{3/2}}e^2\log\left(\frac{2}{\theta_0}\right). \tag{29}$$

Equivalently, $(\eta_{\perp R})^{-1}$ leads to the expression for electrical conductivity. The thermoelectric coefficient is represented by

$$\lambda_R = \frac{3}{16}\left[\left(\frac{2}{\pi}\right)^{1/2}\frac{K_2(z')}{\exp(-z')}\frac{z'^{5/2}}{z'^2 - 0.125z' + 0.07} + 2z^{-1}z'^{-1/2} + z'^{-1/2}\right]z\frac{Nc}{B^2} \tag{30}$$

while the thermal conductivity is as follows:

$$\kappa_R = \frac{1}{6}\left[1 + (1 + 2z^{-1})\left(\frac{\pi}{2}\right)^{1/2}\left(\frac{z'^2 - 0.125z' + 0.07}{z'^{5/2}}\right)\frac{\exp(-z')}{K_2(z')}\right]z \times \left(\frac{\pi}{2}\right)^{1/2}\left(\frac{2m^+}{kT}\right)^{1/2} e^2 \log\left(\frac{2}{\theta_0}\right)\frac{N^2c^2}{B^2}. \quad (31)$$

Equations (29)–(31) essentially reveal that the transport coefficients of interest are largely influenced by z and z', the dimensionless physical quantities that characterize the plasma species in an appropriate moderately-relativistic thermal regime. Note that the expression for electrical conductivity $\eta_{\perp R}^{-1}$ which can be obtained as the reciprocal of (29) is analogous to that obtained by Van Erkelens and Van Leeuwen [15] by putting $n_3 = 0$ in their relevant expression for a ternary plasma based on a lowest-order variational treatment. Such analogy or equivalence is also recognized by Braams and Karney (equation (42), p 1363 of [11]) although they have analysed a Fokker–Planck velocity differential approach. Essentially, though the order of our expression (29) with regard to z' is the same as that of [11, 15], slight numerical mismatch is inevitable owing to differences in collisional formalism or methodology.

Furthermore, it is true that [15] deals with cosmological plasmas and the related formulations concerning diffusion and transport processes, especially in a ternary or quarternary plasma species. To achieve this end, only vectorial bracket forms or integral forms of representations have been shown in respect of electrical conductivity (see abstract, and the relevant equation (3.24) of [15] where on putting $n_3 = 0$, and on employing (D.12), (D.16), (D.17) and (D.18) with some algebra one can recover the relevant formula for a binary plasma).

However, similar expressions can hardly be available in respect to thermal conductivity, e.g. see equations (2.26) and (2.27) in [15] which have been left alone in terms of integral form i.e. $B_{k\gamma}$ and functional form i.e. $Y_{k\gamma}^{(p)}$ representation in a suggestive manner. In fact, no vectorial bracket form has been developed unlike the former.

Quantitatively, if z' approaches the limits, i.e. 1, 10 and 10^2, the corresponding values of $M(z, z')$ approximate to the limits 0.26, 0.826 and 0.98, respectively. Evidently, the above function $M(z, z')$ and its numericals display the nature of the coefficient (electrical resistivity $\eta_{\perp R}$) covered in the approximate temperature limits. Owing to the factor $(kT)^{-3/2}$, the above coefficient markedly diminishes from 1.0×10^{-19} to 5.8×10^{-27} Ω cm as the temperature increases from 10 keV to 1 GeV. Note that we have assumed the magnetic field strength (B) as 10 G and the density of plasma species $N = 10^{13}$ cm^{-3}, where z' lies in the limit 10^2. However, for the strong magnetic field, i.e. $B = 2 \times 10^4$ G, the electrical resistivity decreases from 2.06×10^{-19} to 1.2×10^{-26} Ω cm in the above thermal energy limits. This apparent decrease is caused largely as a result of the rise in temperature and therefore the effect of strong magnetic field included in the modified Coulomb logarithm is negligibly small (see figure 1(a) and 1(b)).

Similarly, when z' takes the limiting values such as 1, 10 and 10^2, the corresponding estimates pertaining to the coefficients λ_R (thermoelectric coefficient) and κ_R (thermal conductivity) register increasing trends with increasing values of z'. As an example, although the latter decreases from 1.4×10^{22} to 8.4×10^{19} cals cm^{-1} s^{-1} K^{-1} for $z' = 1$ and $B = 10$ G as the temperature regime increases from 10 keV to 1 GeV the former remains independent of temperature. Similar variations of κ_R are obvious for different values of z'. However, both the coefficients vary inversely as the square of the magnetic field strength B.

The above results, no doubt, find numerous practical applications in laboratory devices such as RF-heating in tandem mirrors, laser induced heating of plasmas, including fusion

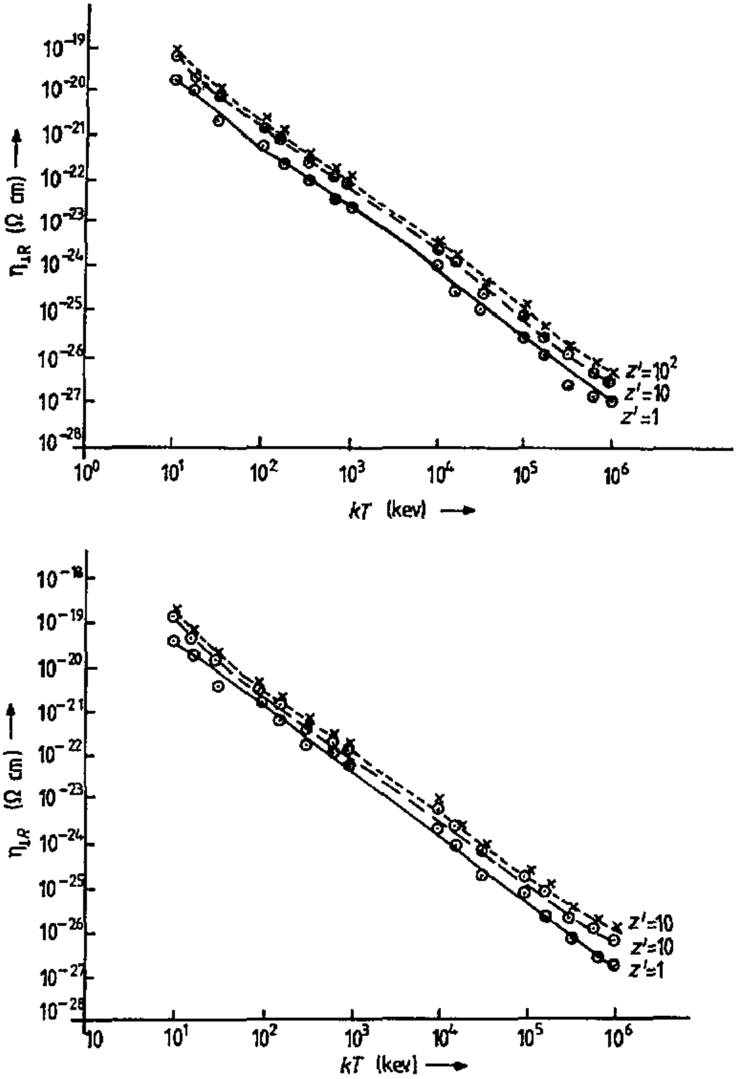

Figure 1. Schematic variation of electrical resistivity $\eta_{\perp R}$ with thermal energy kT in moderately-relativistic fully-ionized magnetoplasma. (*a*) denotes the variation of $\eta_{\perp R}$ for $B = 10$ G; (*b*) denotes the variation of $\eta_{\perp R}$ for $B = 2 \times 10^4$ G (strong field).

pellets like D–T implosion or plasmas in fusion reactors, and the other relativistic plasmas of physical and astrophysical interest.

5. Ultra-relativistic limit

Next, in the ultra-relativistic thermal regimes, we approximate the modified Bessel functions of the second kind for the argument

$$z, z' \ll 1 \qquad \text{as} \qquad K_n(z) \simeq \frac{\Gamma(n)}{2(z/2)^n} \simeq 2^{n-1}\frac{(n-1)!}{z^n}. \tag{32}$$

On further employing the above approximation in equations (12) and (13) we readily obtain

$$M(z,z') \simeq -0.6\left(\frac{\pi}{2}\right)^{1/2} z^{5/2}\log z \tag{33}$$

and

$$R(z,z') \simeq -4.6\left(\frac{\pi}{2}\right)^{1/2} z^{3/2}\log z \tag{34}$$

respectively.

We next combine these values to obtain the transport coefficient relevant to plasmas governed by ultra-relativistic temperature limits, i.e. the electrical resistivity

$$\eta_{\perp\mathrm{R}} = -1.6\left(\frac{\pi}{2}\right) z^{5/2}\log z\frac{m^{-1/2}}{(kT)^{3/2}}e^2\log\left(\frac{2}{\theta_0}\right) \tag{35}$$

the thermoelectric coefficient

$$\lambda = 5.8\frac{Nc}{B^2} \tag{36}$$

and the thermal conductivity

$$\kappa_\mathrm{R} = -2.2\left(\frac{\pi}{2}\right) z^{5/2}\log z\left(\frac{m^+}{kT}\right)^{1/2} e^2\log\left(\frac{2}{\theta_0}\right)\frac{N^2c^2}{B^2}. \tag{37}$$

Quantitatively, if z and z' approach the limits, i.e. 10^{-2} and 10^{-3}, the ultra-relativistic correction factor $z^{5/2}\log z$ correspondingly introduces an estimate to the extent of -2×10^{-5} and -9.5×10^{-8}, respectively; thereby appreciably decreasing the coefficients (35) and (37). The diminishing trend of $\eta_{\perp\mathrm{R}}$ and κ_R in this ultra-relativistic temperature limit is exactly analogous to that of the preceding case, the i.e. moderately-relativistic temperature regime. However, the trend appears to be somewhat more drastic.

In comparison with the preceding moderately-relativistic case the diminishing trends of the physical quantities such as the coefficients $\eta_{\perp\mathrm{R}}$ and κ_R are strikingly more drastic. The electrical resistivity $\eta_{\perp\mathrm{R}}$ decreases from 2.4×10^{-35} to 2.4×10^{-38} Ω cm and the thermal conductivity κ_R sharply diminishes from 8.2×10^{13} to 1.2×10^{9} cal cm^{-1} s^{-1} K^{-1} as kT increases from 10 GeV to 100 GeV with $B = 10$ G.

In summary, we have developed a formalism for the relativistic collisional Boltzmann transport equation in the binary Coulomb collision theory valid for weakly-coupled, fully-ionized, semi-classical plasma. A modified Chapman–Enskog method has been employed to solve the collisional flux integrals and derive relevant expressions for different diffusion transport coefficients in relativistic theory, i.e. the $\boldsymbol{E}\times\boldsymbol{B}$ drift, electrical resistivity, thermoelectric coefficient and thermal conductivity covering all ranges of temperatures. Our expression for electrical conductivity $(\eta_{\perp\mathrm{R}})^{-1}$ does correspond to that of (15) valid for lowest-state variational treatment in an analogous manner. It is identified only in the weakly- or moderately-relativistic temperature regime. The profiles of the above transport coefficients excluding the $\boldsymbol{E}\times\boldsymbol{B}$ drift velocity (which is independent of thermal energy) are graphically displayed (figures 1–3). These coefficients decrease significantly with increasing temperature.

Our generalized and unified model uniquely recovers three limiting cases of approximation, i.e. non-relativistic, weakly- or moderately-relativistic and ultra-relativistic temperature regime limits especially in the giga electron-volt range of energies. Exact calculations in the qualitative and quantitative discussions guarantee that the formalism does extract sufficient data for various observational situations. We believe that our model unravels more fundamental aspects of binary Coulomb interactions in a more

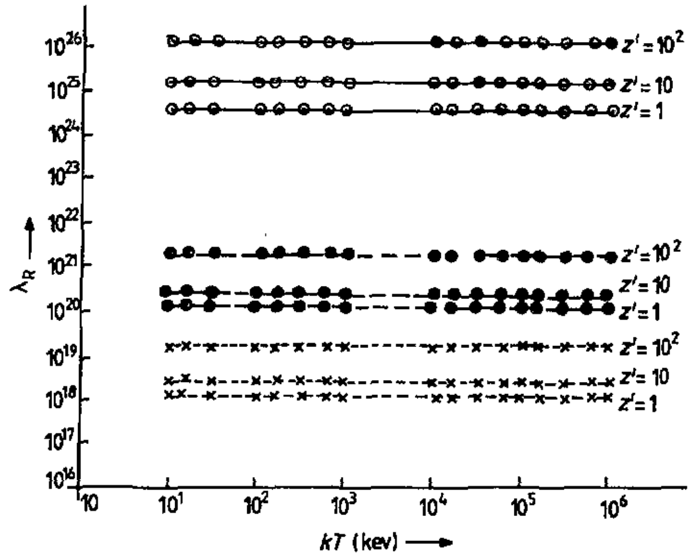

Figure 2. Schematic variation of the thermoelectric coefficient λ_R for different magnetic field strengths in moderately relativistic magnetoplasma. The full curves denote the values of λ_R for $B = 10$ G; the broken curves denote the values for $B = 2 \times 10^3$ G; the dotted curves denote the values for $B = 2 \times 10^4$ G (strong field).

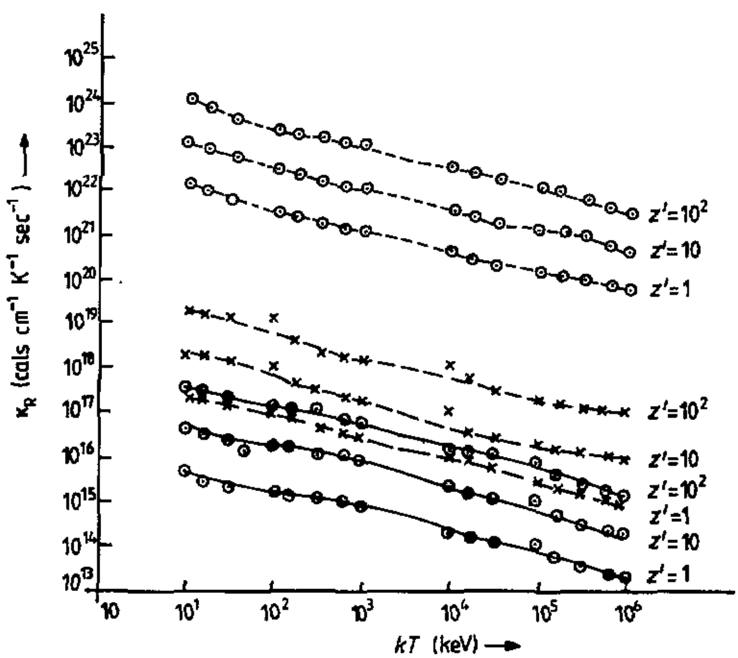

Figure 3. Schematic variation of thermal conductivity κ_R with the thermal energy (kT) in moderately relativistic magnetoplasma. The full curves denote the variation of κ_R for $B = 2 \times 10^4$ G (strong field); the broken curves denote the variation for $B = 2 \times 10^3$ G; the dotted curves denote the variation for $B = 10$ G.

easily accessible manner than any of the earlier works reported so far, and further the model formulates and derives the relevant coefficients explicitly for all the three important temperature regimes for the first time. Thus the paper embodies all the thermal regimes that have significant geophysical and global applications of natural plasmas and the explicit binary interaction phenomena.

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